

Pion Electromagnetic Form Factor at Lower and Higher Momentum Transfer

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The pion electromagnetic form factor is calculated at lower and higher momentum transfer in order to explore constituent quark models and the differences among those models. In particular, the light-front constituent quark model is utilized here to calculate the pion electromagnetic form factor at lower and higher energies. The matrix elements of the electromagnetic current, are calculated with both "plus" and "minus" components of the electromagnetic current in the light-front. Further, the electromagnetic form factor is compared with other models in the literature and experimental data.

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I. INTRODUCTION

Quantum chromodynamics (QCD), is believed as the correct theory of the strong interactions which is one of the four fundamental interactions of nature. One of the most important questions in QCD not yet solved, is the non-perturbative regime. However, with the relativistic constituent quark model (CQM), is possible to obtain answers for hadronic physics in terms of the degrees freedom from the QCD, i.e., quarks and gluons [1]. Before the advent of QCD, the pion, the lowest mass hadronic bound state, providing the long-range attraction part of the nucleon-nucleon potential [2] with the intermediate energies. The main proposal of the light-front approach is to describe consistently the hadronic pion bound state to both, higher and lower momentum transfer regime.

With this purpose, the light-front quantization is utilized to compute the hadronic bound state wave functions [1, 3], which are simpler compared to the instant form quantum field theory [4].

As known, in the light-front models, the bound states wave functions are defined in the hypersurface $x^+ = x^0 + x^3 = 0$, and the wave functions are covariant under kinematical front-form boosts, because the Fock-state decomposition stability [5]. The bound state wave functions with the light-front constituent quark model (LFCQM) have received much attention lately [6, 7].

The quark models show an impressive success in the description of the electromagnetic properties of the hadronic wave functions, for pseudoscalar and spin half particles [8–28] and also vector particles [29–34].

The extraction of the electromagnetic form factor with the light-front approach depends on which component of the electromagnetic current is utilized to calculate the form-factors, due to the problems related with the rotational symmetry breaking and the zero modes, namely a non-valence contribution to the matrix elements of the electromagnetic current [29, 35, 36, 38–40].

It is found in references [29, 30, 37–39, 41, 42] for spin-1 particles, that the plus component of the electromagnetic current, ("J⁺"), is not free from the pair terms contribution (or non-valence contributions) in the Breit frame ($q^+ = 0$), and thus the rotational symmetry is broken.

Then, the matrix elements of the electromagnetic current in the light-front formalism have other contributions for the electromagnetic current besides the valence contribution; that contribution corresponds to the non-valence components or pair terms added to the matrix elements of the electromagnetic current [20, 35, 36, 41] in order to restore the covariance.

If the pair terms contribution is taken correctly, it doesn't matter which component of the electromagnetic current is utilized in the light-front formalism in order to extract the electromagnetic form factors of the hadronic bound states. In the present work, two types of the vertex functions are utilized in order to calculate the pion electromagnetic form-factor for the $\pi - q\bar{q}$ vertex and both are compared with the experimental data [43, 45–50].

At lower momentum transfer, non-perturbative regime of QCD is more important compared with the higher momentum transfer of the perturbative regime of QCD. Perturbative QCD works well over the momentum transfer squared 1.0 (GeV/c)^2 and is predominant around 5.0 (GeV/c)^2 . The studies on light vector and scalar mesons are important, because they give a direction for understanding why QCD works in the non-perturbative regime and also, the light mesons are related with the chiral symmetry breaking.

Hadronic bound states, mesons, are described with other approaches in references [34, 51–64]. Further, another possibility is to study the hadronic bound state with the lattice formulation in the light-front [65].

For the lightest pseudoscalar bound state meson, the models with the Schwinger-Dyson equations [51–53] describe the electromagnetic form factor quiet well, however some differences among the models can be noticed in the literature.

Here, the light-front models for the pion which were presented in previous work [11, 20], are extended to higher momentum transfer and compared with other quark models, for example, the vector meson dominance [68, 69].

This paper is presented in the following; in section II, the model of the wave function for the bound quark-antiquark in the light-front is presented, and the electromagnetic form factor is calculated with non-symmetric and symmetric vertex $\pi - q\bar{q}$, also, in the case of non-symmetric vertex, the plus and minus components of the electromagnetic current are used. As well, in this section, it is presented the calculation for the weak decay constant for the pion.

In section III, the vector dominance model is presented in order to compare with the light-front approach utilized here. Finally, in section IV, the numerical results and discussions are given, and the conclusions are presented in section V.

II. LIGHT-FRONT WAVE FUNCTION AND ELECTROMAGNETIC FORM FACTOR

In the light-front formalism, the main goal is to solve the bound state problem, that is translated in solving the equation below,

$$H_{LF}|\Psi\rangle = M^2|\Psi\rangle. \quad (1)$$

In Eq. (1), the light-front Hamiltonian H_{LF} has the eigenvalues given by the invariant mass M^2 , where the eigenvalues are associated with the physical particles, the eigenstates of the light-front Hamiltonian [1]. The hadronic light-front wave function is related with Bethe-Salpeter wave function (see Ref. [20] for more details about this point). With the light-front wave function it is possible to calculate the matrix elements between hadronic bound states. In the light-front, the meson bound state wave function is a superposition of all Fock states, and the wave function is given by

$$|\Psi_{meson}\rangle = \Psi_{q\bar{q}}|q\bar{q}\rangle + \Psi_{q\bar{q}g}|q\bar{q}g\rangle + \dots \quad (2)$$

With the light-front hadronic wave function above, it is possible to calculate the hadronic electromagnetic form factors from the overlap of light-front wave functions between the final and initial states. In general, the electromagnetic form-factor for the pion is expressed by the covariant equation below,

$$(p + p')^\mu F_\pi(q^2) = \langle \pi(p') | J^\mu | \pi(p) \rangle, \quad q = p' - p, \quad (3)$$

where J^μ is the electromagnetic current, which is possible to be expressed in terms of the quark fields q_f and the charge e (f is the flavor of the quark field): $J^\mu = \sum_f e_f \bar{q}_f \gamma^\mu q_f$. The matrix elements of the electromagnetic current, are written according to the following equation:

$$J^\mu = -i2e \frac{m^2}{f_\pi^2} N_c \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[S(k) \gamma^5 S(k - p') \gamma^\mu S(k - p) \gamma^5 \right] \Gamma(k, p') \Gamma(k, p), \quad (4)$$

where $S(p) = \frac{1}{\not{p} - m + i\epsilon}$ is the quark propagator and $N_c = 3$ is the number of colors, and m is the constituent quark mass. The calculation here is made in the Breit frame, with $p^\mu = (0, -q/2, 0, 0)$ and $p'^\mu = (0, q/2, 0, 0)$ for the initial and final momenta of the system respectively, and the momentum transferred is $q^\mu = (0, q, 0, 0)$ and k^μ is the spectator quark momentum. The factor 2 appears from the isospin algebra [11, 20]. The function $\Gamma(k, p)$, is the regulator vertex function used in the present work to regularize the Feynman amplitude, ie., the triangle diagram for the electromagnetic current, Eq. (4), written above.

Here, we have utilized two possible $\pi - q\bar{q}$ vertex functions; the first one is the non-symmetric vertex, used in the previous work [11, 40]:

$$\Gamma^{(NSY)}(k, p) = \left[\frac{N}{((p - k)^2 - m_R^2 + i\epsilon)} \right], \quad (5)$$

and the second one is, a symmetric vertex, used in the references [20, 26]:

$$\Gamma^{(SY)}(k, p) = \left[\frac{N}{(k^2 - m_R^2 + i\epsilon)} + \frac{N}{((p - k)^2 - m_R^2 + i\epsilon)} \right]. \quad (6)$$

In the expressions for the vertex above, m_R is the regulator mass used in order to keep the amplitudes finite and, also, represents the soft effects at the short range. An important question in QCD and electromagnetic processes is the

current conservation, which is a consequences of the gauge invariance. It is also important to check with the vertex functions, $\Gamma(k, p)$, utilized in the present work, the electromagnetic current conservation. The current conservation is easily proved in the Breit frame (see the reference [36], for this point).

The J^+ component of the electromagnetic current is used to extract the pion electromagnetic form factor from Eq. (3), where the Dirac "plus" matrix is given by $\gamma^+ = \gamma^0 + \gamma^3$. The plus component, $J_\pi^+ (= J^0 + J^3)$, of the electromagnetic current for the pion calculated in the light-front formalism through the triangle Feynman diagram in the impulse approximation, which represents the photon absorption process by the hadronic bound state of the $q\bar{q}$ pair, is given by:

$$\begin{aligned} J_\pi^+ &= 2e(p^+ + p'^+)F_\pi(q^2) \\ &= ie \frac{m^2}{f_\pi^2} N_c \int \frac{dk^- dk^+ d^2 k_\perp}{2(2\pi)^4} \frac{Tr[\mathcal{O}^+] \Gamma(k, p') \Gamma(k, p)}{k^+ (k^- - \frac{f_1 - i\epsilon}{k^+})} \\ &\quad \times \left[\frac{1}{(p^+ - k^+)(p^- - k^- - \frac{f_2 - i\epsilon}{p^+ - k^+})(p'^+ - k^+)(p'^- - k^- - \frac{f_3 - i\epsilon}{p'^+ - k^+})} \right], \end{aligned} \quad (7)$$

where the f_i ($i = 1, 2, 3$) functions above, are defined by, $f_1 = k_\perp^2 + m^2$, $f_2 = (p - k)_\perp^2 + m^2$ and $f_3 = (p' - k)_\perp^2 + m^2$, with the light-front coordinates defined, $a^\pm = a^0 \pm a^3$ and $\vec{a}_\perp = (a_x, a_y)$ [1, 3].

In the expression of the electromagnetic current, Eq.(7), the Jacobian for the transformation to the light-front coordinates is 1/2, and the Dirac trace in the Eq. (7) for the operator \mathcal{O}^+ is written in the light-front coordinates in the Breit frame with Drell-Yan condition ($q^+ = 0$), as:

$$\begin{aligned} Tr[\mathcal{O}^+] &= Tr[(\not{k} + m)\gamma^5(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\gamma^5] \\ &= [-4k^-(k^+ - p^+)^2 + 4(k_\perp^2 + m^2)(k^+ - 2p^+) + k^+ q^2]. \end{aligned} \quad (8)$$

The quadri-momentum integration of the Eq. (7) has two contribution intervals:

(i) $0 < k^+ < p^+$ and (ii) $p^+ < k^+ < p'^+$, where $p'^+ = p^+ + \delta^+$.

The first interval, (i), is the contribution to the valence wave function for the electromagnetic form factor, and the second, (ii), corresponds to the pair terms contribution to the matrix elements of the electromagnetic current. In the case of the non-symmetric vertex with the plus component of the electromagnetic current, the second interval does not give any contribution for the current matrix elements, because the non-valence terms contribution in this case is zero [11, 40].

However, it is not the case for the minus component of the electromagnetic current for the pion, where beyond the valence contribution, we have a non-valence contribution [11] for the matrix elements of the electromagnetic current. For the first interval integration, the pole contribution is $\bar{k}^- = \frac{f_1 - i\epsilon}{k^+}$. After the integration for the light-front energy, k^- , the electromagnetic form factors with nons-symmetric vertex and the plus component of the electromagnetic current is given by

$$\begin{aligned} F_\pi^{+(i)(NSY)}(q^2) &= 2ie \frac{m^2 N^2}{2p^+ f_\pi^2} N_c \int \frac{d^2 k_\perp dk^+}{2(2\pi)^4} \left[\frac{Tr[\mathcal{O}^+]}{k^+ (p^+ - k^+)^2 (p^+ - k^+)^2} \right. \\ &\quad \times \frac{\theta(k^+) \theta(P^+ - k^+)}{(p^- - \bar{k}^- - \frac{f_2 - i\epsilon}{p^+ - k^+})(p^- - \bar{k}^- - \frac{f_3 - i\epsilon}{p^+ - k^+})} \\ &\quad \left. \times \frac{1}{(p^- - \bar{k}^- - \frac{f_4 - i\epsilon}{p^+ - k^+})(P'^- - \bar{k}^- - \frac{f_5 - i\epsilon}{p'^+ - k^+})} \right], \end{aligned} \quad (9)$$

where the Dirac trace above (for the quark on-shell) is:

$$Tr[\bar{\mathcal{O}}^+] = [-4\bar{k}^-(k^+ - p^+)^2 + 4(k_\perp^2 + m^2)(k^+ - 2p^+) + k^+ q^2].$$

The functions f_1 , f_2 and f_3 were already defined and the new functions above are, $f_4 = (p - k)_\perp^2 + m_R^2$ and $f_5 = (p' - k)_\perp^2 + m_R^2$. The light-front wave function for the pion with the non-symmetric vertex is

$$\Psi^{(NSY)}(x, k_\perp) = \left[\frac{N}{(1-x)^2 (m_\pi^2 - \mathcal{M}_0^2)(m_\pi^2 - \mathcal{M}_R^2)} \right], \quad (10)$$

where the fraction of the carried momentum by the quark is $x = k^+/p^+$ and \mathcal{M}_R function is written as

$$\mathcal{M}_R^2 = \mathcal{M}^2(m^2, m_R^2) = \frac{k_\perp^2 + m^2}{x} + \frac{(p - k)_\perp^2 + m_R^2}{(1-x)} - p_\perp^2. \quad (11)$$

In the pion wave function expression, $\mathcal{M}_0^2 = \mathcal{M}^2(m^2, m^2)$ is the free mass operator and the normalization constant N is determined by the condition $F_\pi(0) = 1$.

Finally, the pion electromagnetic form factor expressed with the light-front wave function for the non-symmetric vertex function, is writing as

$$F_\pi^{+(i)(NSY)}(q^2) = \frac{m^2}{p^+ f_\pi^2} N_c \int \frac{d^2 k_\perp dx}{2(2\pi)^3 x} \left[-4 \left(\frac{f_1}{xp^+} \right) (xp^+ - p^+)^2 + 4f_1(xp^+ - 2p^+) \right. \\ \left. + xp^+ q^2 \right] \Psi_f^{*(NSY)}(x, k_\perp) \Psi_i^{(NSY)}(x, k_\perp) \theta(x) \theta(1-x). \quad (12)$$

But in the light-front approach, besides the valence contribution for the electromagnetic current, the non-valence components give another contribution, too [11, 35, 36]. The non-valence components contribution is calculated in the second interval of the integration (ii), through the "dislocation pole method", developed in reference [36]. The non-valence contributions to the electromagnetic form factor in this case, is given:

$$F_\pi^{+(ii)(NSY)}(q^2) = \lim_{\delta^+ \rightarrow 0} 2ie \frac{m^2 N^2}{2p^+ f_\pi^2} N_c \int \frac{d^2 k_\perp dk^+}{2(2\pi)^4} \left[\frac{Tr[\mathcal{O}^+]}{k^+(p^+ - k^+)^2(p^+ - k^+)^2} \right. \\ \left. \times \frac{\theta(p^+ - k^+) \theta(p'^+ - k^+)}{(p^- - \bar{k}^- - \frac{f_2 - i\epsilon}{p^+ - k^+})(p^- - \bar{k}^- - \frac{f_3 - i\epsilon}{p^+ - k^+})(p^- - \bar{k}^- - \frac{f_4 - i\epsilon}{p^+ - k^+})(p'^- - \bar{k}^- - \frac{f_5 - i\epsilon}{p'^+ - k^+})} \right] \\ \propto \delta^+ = 0. \quad (13)$$

As can be seen in equation Eq. (13), the electromagnetic form factor is directly proportional to δ^+ , and that term goes to zero with δ^+ . Then, the non-valence or the pair term contribution for the pion electromagnetic form factor is zero, in the case of non-symmetric vertex for the plus component of the electromagnetic current calculated in the Breit frame [11].

Also, with the minus component of the electromagnetic current, $J_\pi^- (= J^0 - J^3)$, it is possible to extract the pion electromagnetic form factor with non-symmetric vertex Eq. (5). But in this case, we have two contributions, one is the valence contribution for the wave function and the second is the non-valence contribution to the electromagnetic matrix elements of the electromagnetic current [11, 20, 36]. The pion electromagnetic form factor for the minus component of the electromagnetic current, J_π^- , is related with the Dirac matrix by $\gamma^- = \gamma^0 - \gamma^3$, as known in the light-front approach [1, 3]. With the non-symmetric vertex, the minus component of the electromagnetic current is given by,

$$J_\pi^{-(NSY)} = e(p + p')^- F_\pi^{-(NSY)}(q^2) \\ = ie^2 \frac{m^2}{f_\pi^2} N_c \int \frac{d^4 k}{(2\pi)^4} Tr \left[\frac{\not{k} + m}{k^2 - m^2 + i\epsilon} \gamma^5 \frac{\not{k} - \not{p}' + m}{(p' - k)^2 - m^2 + i\epsilon} \gamma^- \right. \\ \left. \times \frac{\not{k} - \not{p} + m}{(p - k)^2 - m^2 + i\epsilon} \gamma^5 \Gamma(k, p') \Gamma(k, p) \right]. \quad (14)$$

The Dirac trace in equation Eq. (14), for the minus component of the electromagnetic current, calculated with the light-front approach, results in the following expression:

$$Tr[\mathcal{O}^-] = [-4k^- k^+ - 4p^+(2k_\perp^2 + k^+ p^+ + 2m^2) \\ + k^- (4k_\perp^2 + 8k^+ p^+ + q^+ + 4m^2)]. \quad (15)$$

In order to calculate the pair terms contribution for the minus component of the electromagnetic current in the second interval integration, ($p^+ < k^+ < p'^+$), the k^- dependence in the trace is performed and the matrix element of the pair terms are written in the equation below:

$$J_\pi^{-(ii)(NSY)} = \lim_{\delta^+ \rightarrow 0} 2ie \frac{m^2}{f_\pi^2} N_c \int \frac{d^2 k_\perp dk^+}{2(2\pi)^4} \left[\frac{Tr[\bar{\mathcal{O}}^-]}{k^+(p^+ - k^+)(p'^+ - k^+)} \right. \\ \left. \times \frac{\theta(p^+ - k^+) \theta(p'^+ - k^+)}{(k^- - \frac{f_1 - i\epsilon}{k^+})(p^- - \bar{k}^- - \frac{f_2 - i\epsilon}{p^+ - k^+})(p^- - \bar{k}^- - \frac{f_4 - i\epsilon}{p^+ - k^+})(p'^- - \bar{k}^- - \frac{f_5 - i\epsilon}{p'^+ - k^+})} \right], \quad (16)$$

where $p'^+ = p^+ + \delta^+$ and $\bar{k}^- = p^- - \frac{f_3 - i\epsilon}{p'^+ - k^+}$. The pair terms contribution for the minus component of the electromagnetic current is obtained with Eq. (16), and the Breit frame is recovered in the limit $\delta^+ \rightarrow 0$:

$$J_\pi^{-(ii)(NSY)} = 4\pi \left(\frac{m_\pi^2 + q^2/4}{p^+} \right) \int \frac{d^2 k_\perp}{2(2\pi)^3} \sum_{i=2}^5 \frac{\ln(f_i)}{\prod_{j=2, i \neq j}^5 (-f_i + f_j)}. \quad (17)$$

The pion electromagnetic form factor with the non-valence contribution is built with the minus component of the matrix elements of the electromagnetic current calculated in Eq. (17):

$$F_{\pi}^{-(ii)(NSY)}(q^2) = \frac{N^2}{2p^-} \frac{m^2}{f_{\pi}^2} N_c \left(4\pi \frac{m_{\pi}^2 + qs^2/4}{p^+} \right) \int \frac{d^2 k_{\perp}}{2(2\pi)^3} \sum_{i=2}^5 \frac{\ln(f_i)}{\prod_{j=2, i \neq j}^5 (-f_i + f_j)}. \quad (18)$$

The full electromagnetic form factor of the pion, is the sum of the partial form factors $F_{\pi}^{-(i)}$ and $F_{\pi}^{-(ii)}$,

$$F_{\pi}^{-(NSY)}(q^2) = \left[F_{\pi}^{-(i)(NSY)}(q^2) + F_{\pi}^{-(ii)(NSY)}(q^2) \right]. \quad (19)$$

If the pair terms are not taken into account, the rotational symmetry is broken and the covariance is lost for the J_{π}^{-} component of the electromagnetic current, as can be seen in Fig. 1. After the pair terms or zero modes contribution add in the calculation of the electromagnetic form factor with the minus component of the electromagnetic current, the following identity is obtained,

$$F_{\pi}^{-(NSY)}(q^2) = F_{\pi}^{+(NSY)}(q^2), \quad (20)$$

and the full covariance is restored.

In the next step, it is employed the symmetric vertex $\pi - q\bar{q}$ with the plus component, "+", of the electromagnetic current, Eq. (6), as utilized in reference [20].

This vertex is symmetric by the exchange of the quadri-momentum of the quark and the anti-quark. In the light-front coordinates it is written as,

$$\begin{aligned} \Gamma(k, p) = & \mathcal{N} \left[k^+ \left(k^- - \frac{k_{\perp}^2 + m_R^2 - i\epsilon}{k^+} \right) \right]^{-1} + \\ & \mathcal{N} \left[(p^+ - k^+) \left(p^- - k^- - \frac{(p - k)_{\perp}^2 + m_R^2 - i\epsilon}{p^+ - k^+} \right) \right]^{-1}. \end{aligned} \quad (21)$$

With the symmetric vertex, the pion valence wave function results in the expression

$$\begin{aligned} \Psi^{(SY)}(x, \vec{k}_{\perp}) = & \\ & \left[\frac{\mathcal{N}}{(1-x)(m_{\pi}^2 - \mathcal{M}^2(m^2, m_R^2))} + \frac{\mathcal{N}}{x(m_{\pi}^2 - \mathcal{M}^2(m_R^2, m^2))} \right] \frac{p^+}{m_{\pi}^2 - M_0^2}. \end{aligned} \quad (22)$$

The electromagnetic form factor for the pion valence wave function, the expression above, calculated in the Breit frame ($q^+ = 0$), is

$$\begin{aligned} F_{\pi}^{(SY)}(q^2) = & \frac{m^2 N_c}{p^+ f_{\pi}^2} \int \frac{d^2 k_{\perp}}{2(2\pi)^3} \int_0^1 dx \left[k_{on}^- p^{+2} + \frac{1}{4} x p^+ q^2 \right] \\ & \times \frac{\Psi_f^{*(SY)}(x, k_{\perp}) \Psi_i^{(SY)}(x, k_{\perp})}{x(1-x)^2}, \end{aligned} \quad (23)$$

where $k_{on}^- = (k_{\perp}^2 + m^2)/k^+$ and the normalization constant \mathcal{N} is determined from the condition $F_{\pi}^{SY}(0) = 1$. The pion electromagnetic form factor calculated with the symmetric wave function is presented the Fig. 1 for higher momentum, and in Fig. 2 for low momentum transfer. In both regions, the differences between the symmetric and non-symmetric vertex are not so large.

The pion decay constant, measured in the weak leptonic decay, is given, with the partial axial current conservation by, $P_{\mu} < 0 | \bar{q} \gamma^{\mu} \gamma^5 \tau_i q / 2 | \pi_j > = i m_{\pi}^2 \delta_{ij}$ [11, 20], and with the the vertex function $\Gamma(k, p)$, is written by,

$$i f_{\pi} P^2 = N_c \frac{m}{f_{\pi}} \int \frac{d^4 k}{(2\pi)^4} Tr [\not{p} \gamma^5 S(k) \gamma^5 S(k-p)] \Gamma(k, p). \quad (24)$$

In the case of the symmetric and non-symmetric vertices, (see Eqs. (5) and (6)), the expressions for the decay constant are, respectively,

$$f_{(\pi)}^{(SY)} = \frac{m^2 N_c}{f_\pi} \int \frac{d^2 k_\perp dx}{4\pi^3 x(1-x)} \Psi_\pi^{(SY)}(x, \vec{k}_\perp; m, \vec{0}) , \quad (25)$$

and

$$f_{(\pi)}^{(NSY)} = \frac{m^2 N_c}{f_\pi} \int \frac{d^2 k_\perp}{4\pi^3} \frac{dx}{x} \Psi_\pi^{(NSY)}(k^+, \vec{k}_\perp; m, \vec{0}) . \quad (26)$$

In the numerical calculations, (see the results section), the obtained values of the decay constant with the expressions above, for both models of light-front calculations, do not have significant discrepancies.

In Next section, the vector meson dominance is presented.

III. VECTOR MESON DOMINANCE

In the 1960's, Sakurai [66, 67] proposed the theory of *Vector Meson Dominance* (VMD); a theory of strong interactions with the local gauge invariance, mediated by vector mesons and based on the non-Abelian field theory of Yang-Mills. However, it is possible to have two lagrangian formulations of the vector meson dominance, the first was introduced by Kroll, Lee and Zumino [68] and is customary called VMD-1. The pion electromagnetic form factor calculated with this formulation of the vector meson dominance, results in:

$$F_\pi^{VMD1}(q^2) = \left[1 - \frac{q^2}{q^2 - m_\rho^2} \frac{g_{\rho\pi\pi}}{g_\rho} \right] . \quad (27)$$

This equation for the electromagnetic form factor satisfies the condition $F_\pi(0) = 1$, independent of assumption about the coupling constants, $g_{\rho\pi\pi}$ and g_ρ .

In the second formulation of the vector meson dominance, the Lagrangian has a photon mass term, and the photon propagator has a non-zero mass; that version is usually called VMD-2. With this second formulation of the vector meson dominance, the pion electromagnetic form factor is written:

$$F_\pi^{VMD2}(q^2) = \left[-\frac{m_\rho^2}{q^2 - m_\rho^2} \frac{g_{\rho\pi\pi}}{g_\rho} \right] . \quad (28)$$

In the equation above, it is necessary that, the condition $F_\pi(0) = 1$ is satisfied, only if the universality limit is taken into account, or, translate in the following equality, $g_{\rho\pi\pi} = g_\rho$. In the universality limit, like advocate by J. Sakurai, the two formulations of the vector meson dominance are equivalent.

For the present work, in Eq. (27) and Eq. (28), the rho meson mass utilized is the experimental value, $m_\rho = 0.767$ GeV, and, from the universality, $g_{\rho\pi\pi} = g_\rho$, the results at zero momentum for both equations satisfy $F_\pi(0) = 1$.

In the present case here, only the lightest vector resonance rho meson is taken account in the monopole model of the VMD as can be seen in Eq. (27) or Eq.(28). The vector meson dominance works quite well in the timelike region below the $\pi\pi$ threshold. At low energies for the space-like region, the vector meson dominance model gives a reasonable description for the pion electromagnetic form factor. For more details and results about the vector meson dominance, see references [69–71].

IV. RESULTS

The pion electromagnetic form factor, presented consistently with previous and later works, are extended at higher momentum transfer region, running $Q^2 = -q^2$ up to 20 (GeV/c)². Here, the models of the $\pi - q\bar{q}$ vertices, i.e, non-symmetric and symmetric vertices [11, 20] are compared with the vector meson dominance (VMD), and shown in Figs. 1, 2 and 3, for low and higher low momentum transfer. The pion electromagnetic radius, is calculated with the derivative of the electromagnetic form factor for the pion, $\langle r^2 \rangle = -6dF(q^2)/dq^2|_{q^2 \simeq 0}$, for both model of the vertices presented here.

In the case of the non-symmetric vertex, the pion radius is utilized to fix the parameters of the model. The parameters are the quark mass $m_q = 0.220$ GeV and the regulator mass $m_R = 1.0$ GeV. The pion mass utilized is the experimental value, $m_\pi = 0.140$ GeV. The experimental radius of the pion is $r_{exp} = 0.672 \pm 0.02$ fm [43, 44].

Using the pion decay constant calculation in the non-symmetric vertex model and with the parameters above, the pion decay constant obtained is $f_\pi = 92.13$ MeV, which is close to the experimental value, $f_\pi \simeq 92.10$ [44].

In the case of the symmetric vertex, the parameters are the quark mass $m_q = 0.220$ GeV, the regulator mass $m_R = 0.60$ GeV and the experimental mass of the pion, $m_\pi = 0.140$ GeV.

Our choice for the regulator mass, fits the pion decay constant, $f_\pi^{exp} = 92.1$ MeV, for the symmetric vertex, quite well compared with the experimental data [44].

Both light-front models, with symmetric and non-symmetric vertex, have good agreement with the experimental data at low energy, however, some differences are noticeable in the region $Q^2 \geq 1.0$ (GeV/c)² (see the fig. 1). The experimental data collected from reference [46] are described well up to 10 (GeV/c)² with both the symmetric and non-symmetric vertex functions. For the minus component of the electromagnetic current, J^- , the pair terms or non-valence components of the electromagnetic current contributions, are essential to obtain the full covariant pion electromagnetic form factor, and to the end to respect the covariance.

TABLE I: Results for the low-energy electromagnetic π -meson observables with light-front and another models .

Model	r_π (fm)	f_π (MeV)	$r_\pi \cdot f_\pi$
Sym. (LF)	0.740	92.40	0.346
Non-Sym.(LF)	0.679	93.10	0.320
LFBS Model [25]	0.651	91.91	0.304
Dyson-Sch. [74]	0.550	92.0	0.256
KLZ Model [75]	0.631	-	-
Exp. [44]	0.672 ± 0.02	92.1	0.314 ± 0.010

Constituent quark models formulated with the light-front approach presented here, give a good agreement with the experimental data [43, 45–47, 49, 50]. The ratios between the electromagnetic current in the light-front and the electromagnetic current calculated in the instant form, are given by the following equations,

$$\begin{aligned}
 Ra^I &= \frac{J_{LF}^+}{J_{Cov}^+}, & Ra^{II} &= \frac{J_{LF}^-}{J_{Cov}^-}, \\
 Ra^{III} &= \frac{J_{LF}^- + J_{LF}^{-(Pair)}}{J_{Cov}^-}, & Ra^{IV} &= \frac{J_{LF}^-}{J_{Cov}^+}, \\
 Ra^V &= \frac{J_{LF}^- + J_{LF}^{-(Pair)}}{J_{Cov}^+},
 \end{aligned} \tag{29}$$

where the non-symmetric vertex is utilized according to Eq. (5).

In Eq. (29), above, Ra^I is the plus component of the electromagnetic current calculated in the light-front divided that of the instant form formalism, since the pair terms do not give contribution for the plus component of the electromagnetic current, so the ratio Ra^I is constant (see Fig. 4). The second ratio, Ra^{II} , is the minus component of the electromagnetic current, J^- , calculated with the light-front formalism and divided by the electromagnetic current calculated in the instant form. In Ra^{III} ratio, the pair terms contribution to the electromagnetic current is included, so the covariance is restored.

The ratios Ra^{IV} and Ra^V are the "minus" components of the electromagnetic current without and with the pair terms contribution, respectively, divided by the "plus" component of the electromagnetic current calculated in the instant form formalism.

As can be seen in Fig. 4, the rotational symmetry in the light-front formalism is broken, because the pair terms or non-valence contribution for the electromagnetic current is not taken into account properly. The restoration of the symmetry breaking is obtained by adding the pair terms contribution to the minus component of the electromagnetic current calculated in the light-front.

Experimental data for $Q^2 \gtrsim 1.5$ (GeV/c)² for the pion electromagnetic form factor (see the Fig. 1), is not precise in order to make a decision satisfactorily among the phenomenological models to select a best description for the pion elastic electromagnetic form factor nor in the end, the correct pion wave function.

We define the following equations, in order to compare the magnitude of the breaking of the rotational symmetry for the pion electromagnetic form factor calculation with the light-front models, and vector meson dominance model,

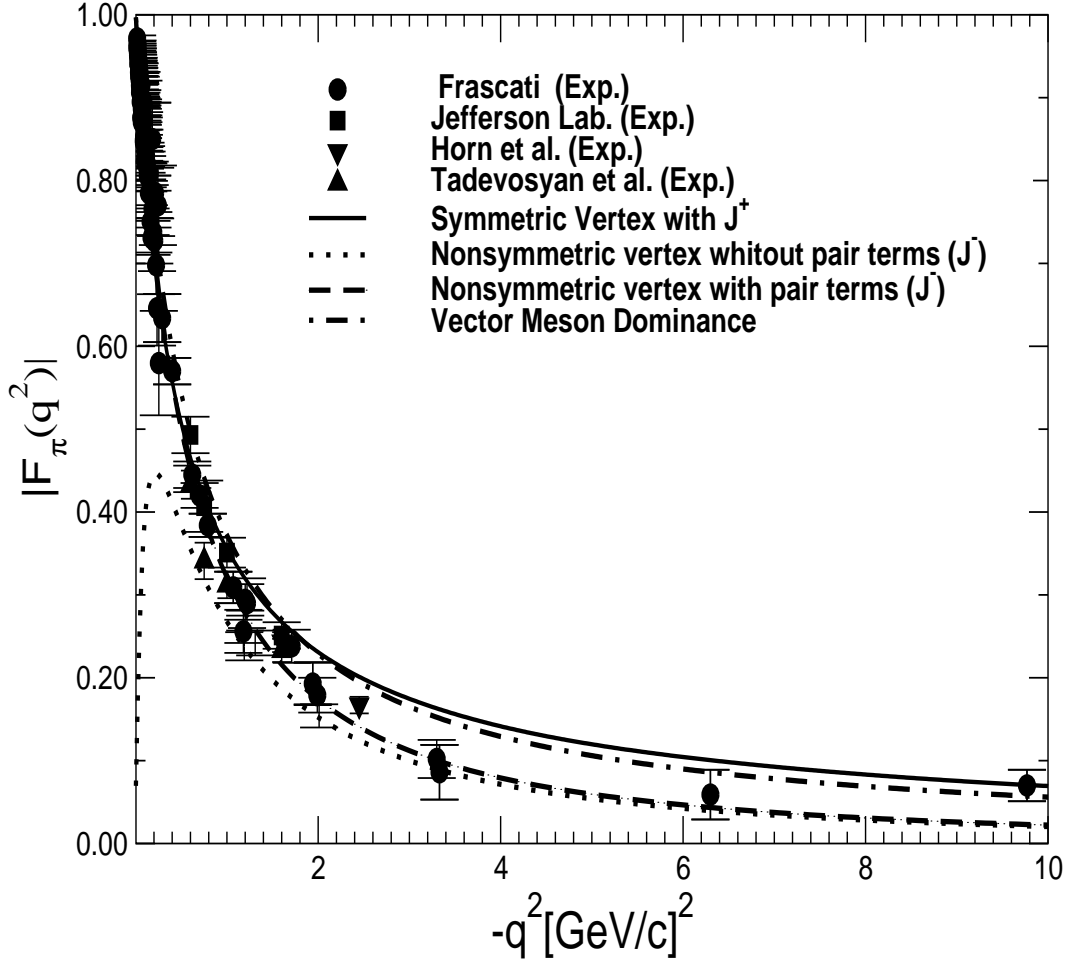


FIG. 1: Pion electromagnetic form factor calculated with light-front constituent quark model, for the plus and minus components of electromagnetic current, compared with experimental data and vector meson dominance. Data are from [46, 47, 49, 50]. Solid line is the full covariant form factor with J_π^+ (symmetric vertex for the $\pi - q\bar{q}$). The dashed line is line the form factor with J_π^- plus pair terms contribution, and the dotted line is the pion form factor without the pair terms contribution with the minus component of the electromagnetic current, where both curves are with the nonsymmetric vertex. After added the non-valence contribution, the pion electromagnetic form factor calculated with the plus or minus component of the electromagnetic current give the same results for the nonsymmetric vertex.

and, also with the covariant calculations; the strategy, is that we try to amplify the differences among theoretical models and experimental data:

$$\begin{aligned}
 \Delta_1 &= \left[q^2 F_\pi^{(VMD)}(q^2) - q^2 F_\pi^{+(NSY)}(q^2) \right], \\
 \Delta_2 &= \left[q^2 F_\pi^{(COV)}(q^2) - q^2 F_\pi^{-(i)(NSY)}(q^2) \right], \\
 \Delta_3 &= \left[q^2 F_\pi^{(COV)}(q^2) - q^2 F_\pi^{-(i+ii)(NSY)}(q^2) \right], \\
 \Delta_4 &= \left[q^2 F_\pi^{(VMD)}(q^2) - q^2 F_\pi^{(exp)}(q^2) \right].
 \end{aligned} \tag{30}$$

The results of the calculations above are shown in the Figs. 5 and 6 at low and higher momentum transfer for the models presented here. The results in Fig. 5, confirm the validity of the vector meson dominance model at very low momentum transfer ($Q^2 \leq 0.5 \text{ (GeV/c)}^2$).

But, for $Q^2 > 0.5 \text{ (GeV/c)}^2$ (see Fig. 6), the discrepancies between the vector meson dominance model, the light-

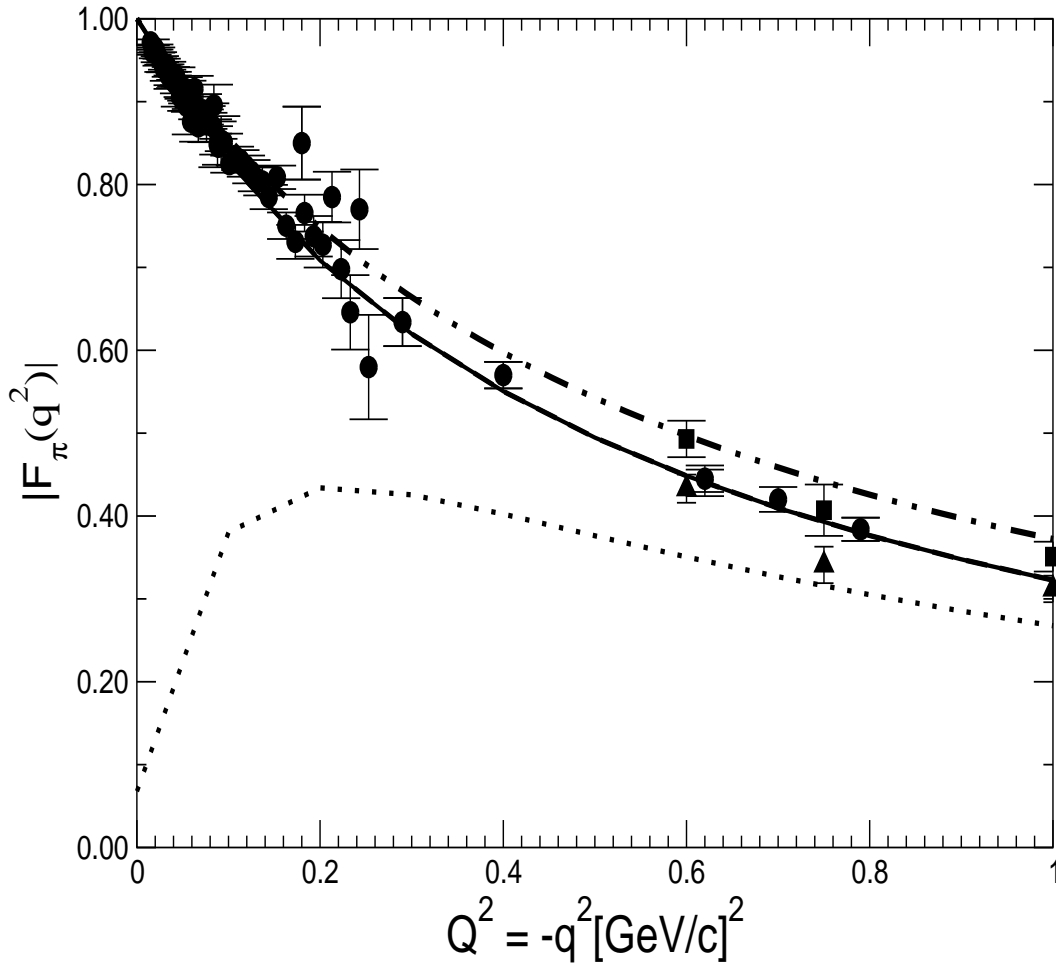


FIG. 2: Pion electromagnetic form factor for small Q^2 . Labels are the same as those in Fig. 1. Experimental data are from Ref. [45–47].

front models and experimental data are more emphasized. In the case of Δ_3 , (see the definition above in the text), the covariance is respected exactly, because the difference is zero in the interval integration sum, [(i)+(ii)], for the J^- component of the electromagnetic current.

The electromagnetic form factor for the pion calculated with the matrix elements of the electromagnetic current gives the same results as the electromagnetic form factor of the pion calculated with usual covariant quantum field theory [4].

For the higher momentum transfer, the asymptotic behavior for the wave function of the non-symmetric vertex model produce $q^2 F_\pi 15 \text{ (GeV/c)}^2 \approx 0.18 \text{ (GeV/c)}^2$. That result is compared with the leading-order-perturbative QCD, $Q^2 F_\pi(Q^2) \approx 0.15 \text{ (GeV/c)}^2$, for $\alpha_s(Q^2 = 10 \text{ (GeV/c)}^2) \approx 0.3$ and with Dyson-Schwinger approach, $Q^2 F_\pi(Q^2) \approx 0.12 - 0.19 \text{ (GeV/c)}^2$, for momentum transfer between $Q^2 \approx 10 - 15 \text{ (GeV/c)}^2$ [74].

V. CONCLUSIONS

In the present work, the electromagnetic form factor of the pion was investigated in the range $0 < Q^2 < 20 \text{ (GeV/c)}^2$ with light-front constituent quark model. The light-front formalism is known nowadays as a natural way to describe the systems with relativistic bound state, like the pion. With this approach it is possible to calculate the electromagnetic form factors in a most suitable way.

However, problems related with the broken of the rotational symmetry in the light-front formalism are important

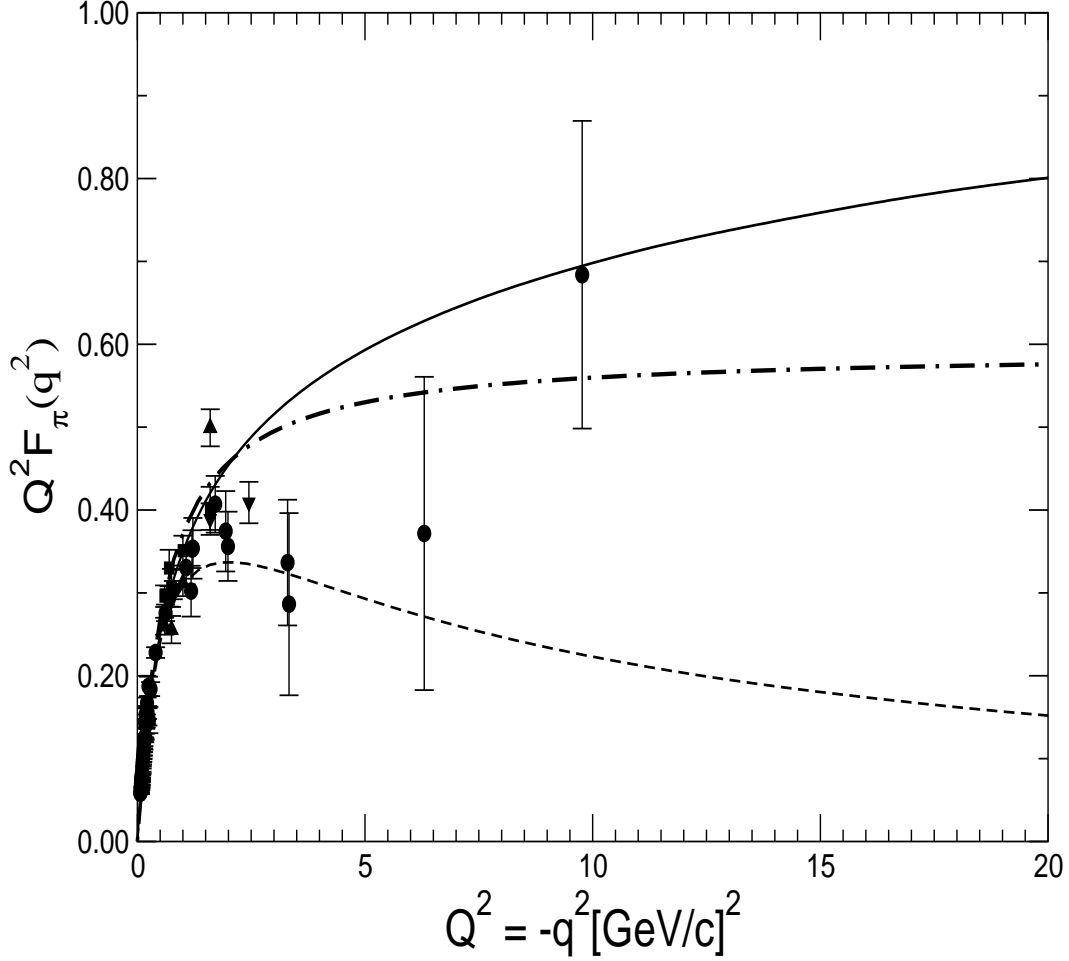


FIG. 3: Pion electromagnetic form factor for higher Q^2 . Labels are the same as those in Fig. 1.

and the pair terms or no-valence terms contribution for the covariance restoration in higher energies is also necessary to be taken care of [11, 29].

After adding the pair terms, or non-valence components in the matrix elements of the electromagnetic current, the covariance is completely restored, and it doesn't matter which component of the electromagnetic current, J^+ or J^- , is utilized in order to extract the pion form factor with the light-front approach, as can be seen in Figs. 1, 2 and 3.

In terms of the electromagnetic current, the numerical results in Fig.4, show the importance of the non-valence components for the electromagnetic current and the dependence which component of the currents utilized at low momentum transfer, the inclusion of the non-valence components of the electromagnetic current is essential for the minus component of the electromagnetic current, to give the full covariance.

In Eq.(29) the ratios Ra^I , Ra^{III} and Ra^V , produce constant values, but the ratios Ra^{II} and Ra^{IV} are not, because the non-valence components of the electromagnetic current is not included in the light-front approach calculation (see Fig 4).

The comparison between the light-front models for the vertex $\pi - q\bar{q}$ with other hadronic models for the pion electromagnetic form-factor has a good agreement between them, however, some differences arise between these models when energies are in the higher region, $Q^2 \gtrsim 2$ (GeV/c) 2 .

With Eq.(30), the differences between the models analyzed in the present work are clear, for lower and higher momentum transfer, because the set of equations increase the possible differences among the models presented here.

Since the pion electromagnetic form-factor is sensitive to the model utilized, it is important to compare different models including new experimental data, and to extract new information about the sub-hadronic structure of the pion bound state.

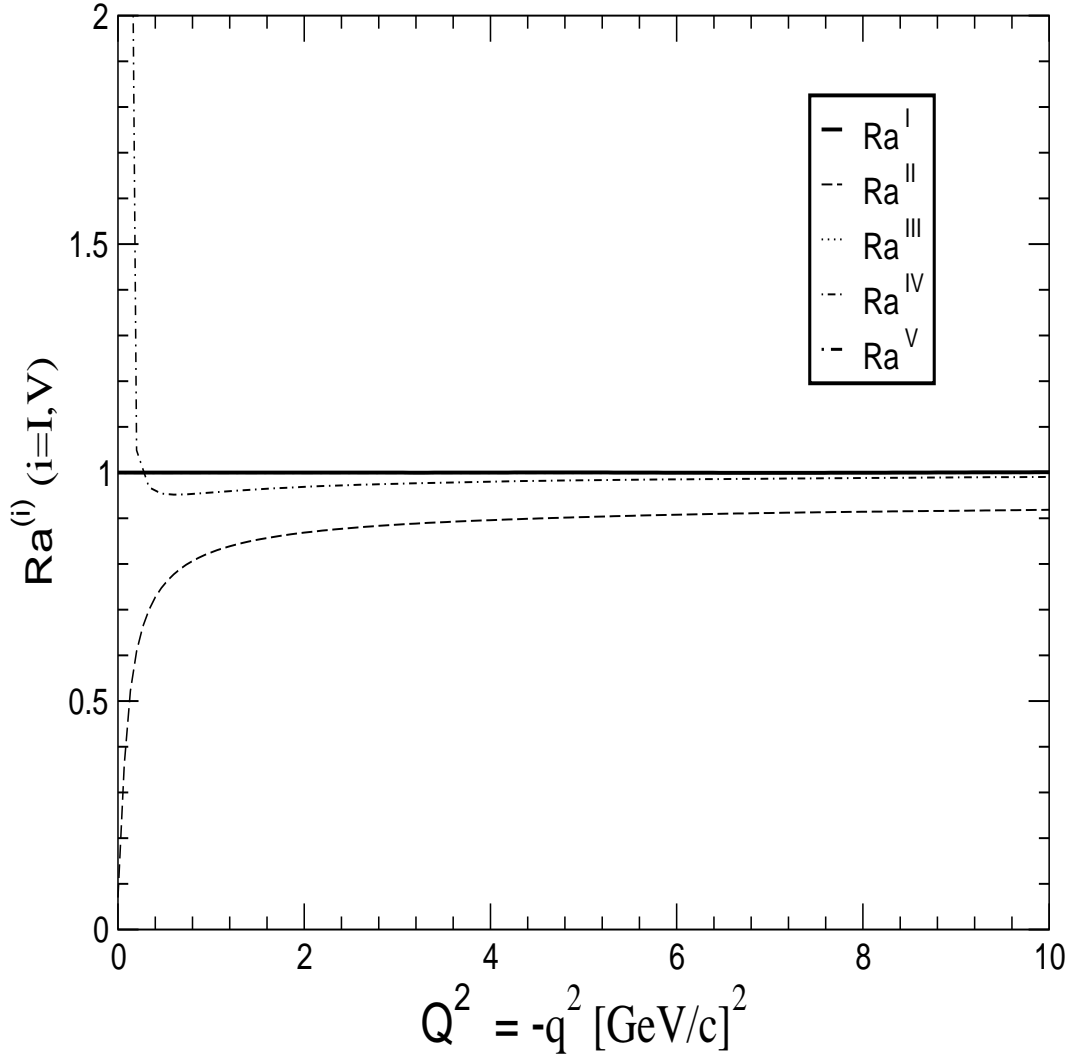


FIG. 4: Pion electromagnetic current ratios, see Eq.(29) in the text.

The light-front approach is a good framework to study the pion electromagnetic form factor. However, the inclusion of the non-valence components of the electromagnetic current is essential for both low and higher momentum transfer.

To conclude, the light-front formalism and the vertex models for $\pi - q\bar{q}$ utilized in the present work with symmetric and non-symmetric vertices, can describe the new experimental data for the pion electromagnetic form factor with very good agreement. In the next step, the calculations for the vector mesons, like ρ -meson and vector kaon, are in progress in order to compare the light-front constituent models with the other models.

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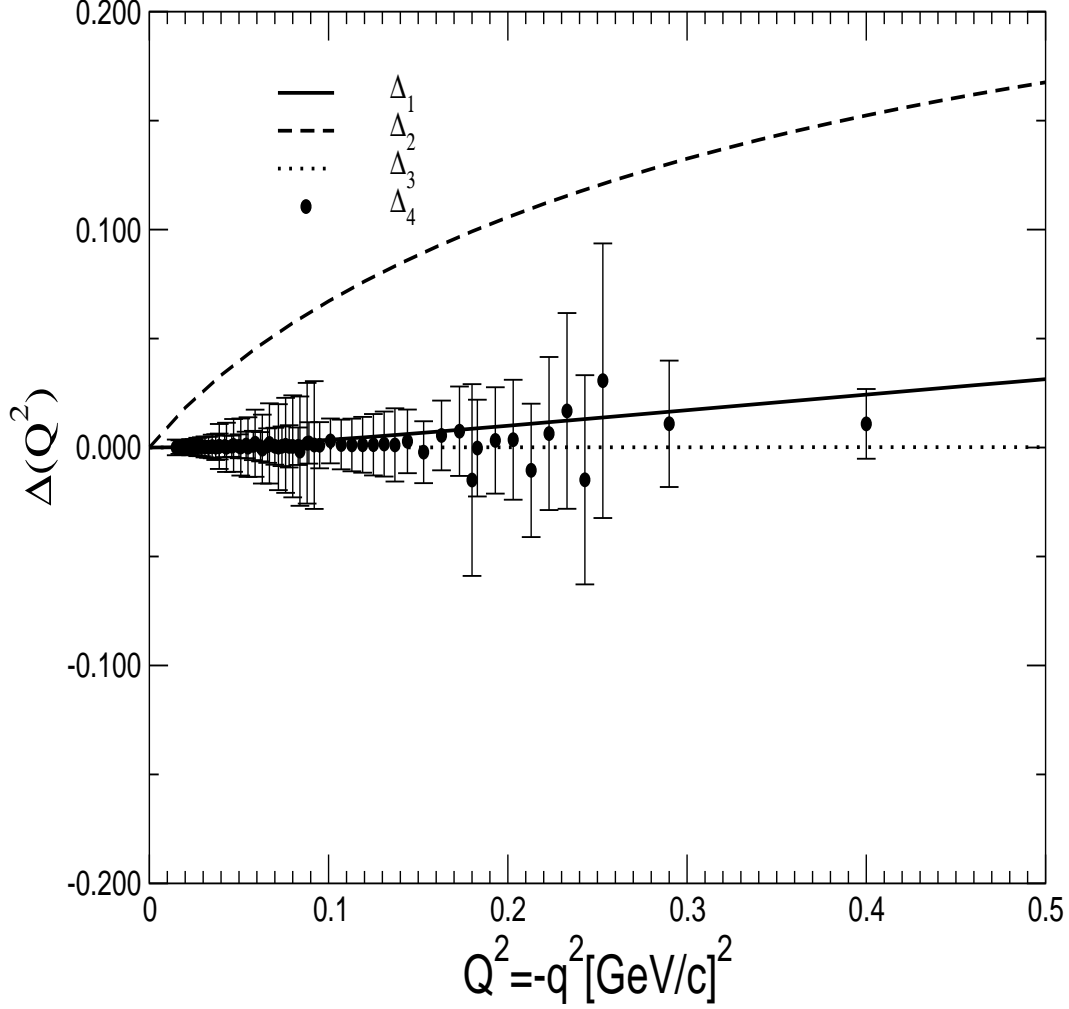


FIG. 5: Figure labels are the same as those in Eq.(30). The range for the momentum transfer given here is up to 0.5 (GeV/c)².

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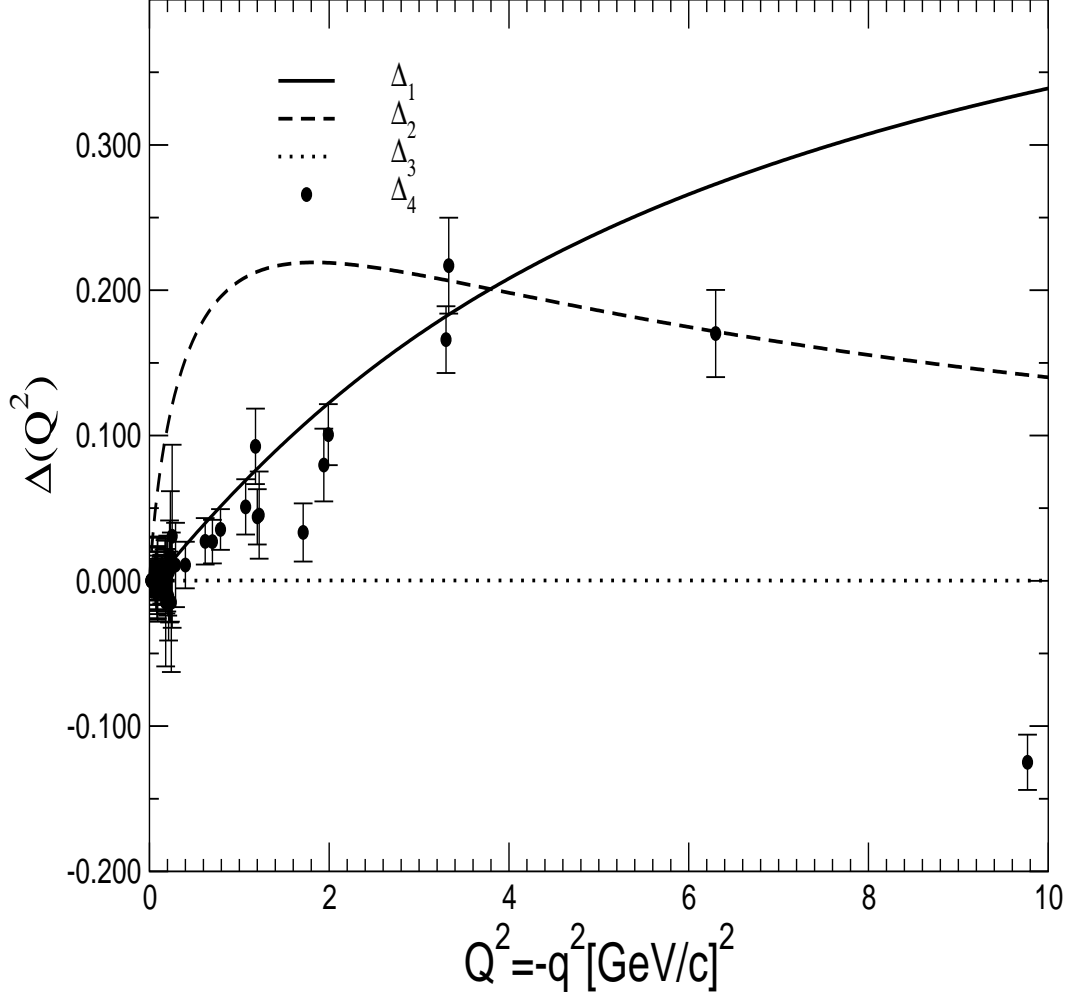


FIG. 6: Figure labels are the same as those in figure 4 (see also Eq.(30), here the momentum transfer is up to 10 (GeV/c)².

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